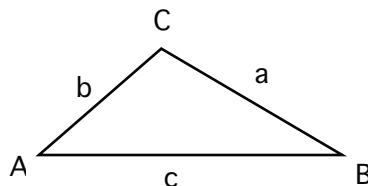


LAW OF SINES AND LAW OF COSINES

#10

LAW OF SINES is used to solve for the missing parts of any triangle determined by ASA or AAS. For any $\triangle ABC$, it is always true that:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Example 1 ASA or AAS

If $m\angle A = 53^\circ$, $a = 18'$, and $m\angle B = 39^\circ$, you can solve for b by substituting the values and solving the equation. Remember that the sine values represent numbers in the equation.

$$\frac{\sin 53^\circ}{18} = \frac{\sin 39^\circ}{b}$$

Substitute.

$$b(\sin 53^\circ) = 18(\sin 39^\circ)$$

Fraction Busters.

$$b = \frac{18(\sin 39^\circ)}{\sin 53^\circ}$$

Solving for b .

$$b \approx 14.18'$$

Calculations.

Note that you can use your calculator to convert the sine values into decimal approximations at any time during the solution, but it is most efficient to wait until the end of the problem.

LAW OF COSINES is used to solve for the missing parts of any triangle determined by SSS or SAS. For any $\triangle ABC$, it is always true that:

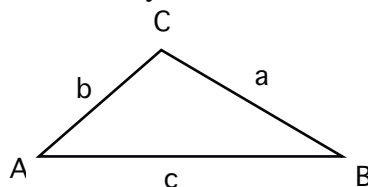
$$a^2 = b^2 + c^2 - 2bc \cos A$$

or

$$b^2 = a^2 + c^2 - 2ac \cos B$$

or

$$c^2 = b^2 + a^2 - 2ba \cos C$$



Example 2 SAS

If $m\angle B = 38^\circ$, $c = 25'$, and $a = 18'$, you can solve for b by substituting the values and solving the equation.

$$b^2 = 25^2 + 18^2 - 2(25)(18) \cos 38^\circ \quad \text{Substitute.}$$

$$b^2 = 625 + 324 - 900(0.7880) \quad \text{Evaluate.}$$

$$b^2 \approx 239.79 \quad b \approx 15.49' \quad \text{Simplify}$$

Example 3 SSS

If $c = 18'$, $a = 5'$, and $b = 16'$ then you can solve for $\angle A$ by substituting the values and solving the equation.

$$5^2 = 16^2 + 18^2 - 2(16)(18) \cos A \quad \text{Substitute.}$$

$$25 = 256 + 324 - 576 \cos A \quad \text{Simplify}$$

$$25 = 580 - 576 \cos A \quad \text{Simplify}$$

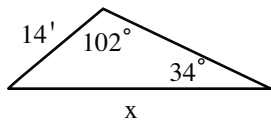
$$0.9635 = \cos A \quad \text{Solve for } \cos A$$

$$15.52^\circ \approx A \quad \text{Use } \cos^{-1} \text{ on calculator.}$$

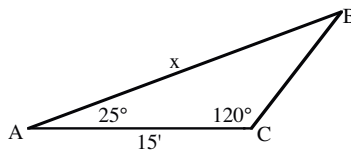
Note: With SSA triangles it is possible to have zero, one, or two solutions

Use the law of sines or the law of cosines to find the required part of the triangle.

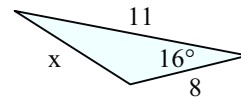
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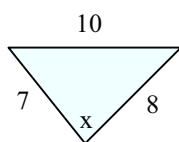
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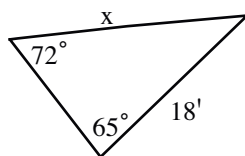
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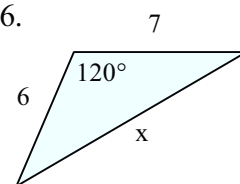
4.



5.



6.



Draw and label a triangle similar to the one in the examples. Use the given information to find the required part(s).

- | | |
|--|---|
| 7. $m\angle A = 40^\circ$, $m\angle B = 88^\circ$, $a = 15$.
Find b . | 8. $m\angle B = 75^\circ$, $a = 13$, $c = 14$.
Find b . |
| 9. $m\angle B = 50^\circ$, $m\angle C = 60^\circ$, $b = 9$.
Find a . | 10. $m\angle A = 62^\circ$, $m\angle C = 28^\circ$,
$c = 24$. Find a . |
| 11. $m\angle A = 51^\circ$, $c = 8$, $b = 12$.
Find a . | 12. $m\angle B = 34^\circ$, $a = 4$, $b = 3$.
Find c . |
| 13. $a = 9$, $b = 12$, $c = 15$.
Find $m\angle B$. | 14. $m\angle B = 96^\circ$, $m\angle A = 32^\circ$, $a = 6$.
Find c . |
| 15. $m\angle C = 18^\circ$, $m\angle B = 54^\circ$, $b = 18$.
Find c . | 16. $a = 15$, $b = 12$, $c = 14$.
Find $m\angle C$. |
| 17. $m\angle C = 76^\circ$, $a = 39$, $b = 19$.
Find c . | 18. $m\angle A = 30^\circ$, $m\angle C = 60^\circ$, $a = 8$.
Find b . |
| 19. $a = 34$, $b = 38$, $c = 31$.
Find $m\angle B$. | 20. $a = 8$, $b = 16$, $c = 7$.
Find $m\angle C$. |
| 21. $m\angle C = 84^\circ$, $m\angle B = 23^\circ$, $c = 11$.
Find b . | 22. $m\angle A = 36^\circ$, $m\angle B = 68^\circ$,
and $b = 8$. Find a and c . |
| 23. $m\angle B = 40^\circ$, $b = 4$, and $c = 6$.
Find a , $m\angle B$, and $m\angle C$. | 24. $a = 2$, $b = 3$, $c = 4$.
Find $m\angle A$, $m\angle B$, and $m\angle C$. |